# Extra Counting Practice Problems

Statistics for Economics - ECON 30330

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Spring 2017

## Problems

1. How many 6 digit plate numbers can be made with numbers 0 to 6 and letters A, B, and C,
   (a) such that the plate number always starts with two letters and ends with a number?
   (b) such that the plate number always starts with two letters and ends with a number, and repetition is not allowed?
2. How many ways can we seat 2 women, 3 kids, and 2 men on a row of 7 numbered chairs, such that the three kids seat near each other and near at least one woman at all times?
3. How many 6 digit passwords can be generated with digits 0 to 9 and letters A to Z,
   (a) such that the password always contains at least 2 letters?
   (b) such that the password always starts with at least 2 letters?
   (c) such that the password always starts with at least 2 letters and repetition is not allowed?
4. How many even integers are there that are greater than 99 and less than or equal to 400?
5. How many even integers are there that are greater than 99 and less than or equal to 400, with digits 0, 1, 2, 3, 4, and 5?
6. A group of college students consisting of 20 freshmen, 10 sophomores, and 2 juniors are participating in a competition. There are going to be winning awards for the first three

winners, a gold medal for the 1st place, a silver medal for the second place, and a bronze medal for the third place.

   (a) What is the probability that all winners are from the same class?
   (b) What is the probability that all juniors are among the winners?
   (c) What is the probability that one student from each class wins a medal?

7. There are 10 female and 8 male candidates for a committee of 4 individuals.
   (a) What is the probability that all the elected candidates are female?
   (b) What is the probability that two female and two male candidates get elected?
   (c) What is the probability that at least two male candidates get elected?

8. How many unique ways are there to arrange the letters in the word PRIOR?

9. In how many different ways can the letters of the word CORPORATION be arranged so that the vowels always come together?

10. In how many different ways can the letters of the word MATHEMATICS be arranged so that the vowels always come together?

11. How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated?

## Suggested Solutions

1. There are a total of 7 numbers and 3 letters to choose from

   (a) There are 3 options for the first digit, 3 options for the second digit, and 7 options for the last digit, and the three digits in the middle can be any of the numbers or letters, so there are 10 options for each. So, a total possible plate numbers of $3 \times 3 \times 10 \times 10 \times 10 \times 7$.

   (b) There are 3 options for the first digit, 2 options for the second digit, and 7 options for the last digit, and the three digits in the middle can be any of the numbers or letters except for what we've already used for the 1st, 2nd, and 6th places, so there are 7 options 3rd place, 6 options for 4th place, and 5 options for the 5th place. So, a total possible plate numbers of $3 \times 2 \times 7 \times 6 \times 5 \times 7$.

2. The 3 kids and one of the 2 women are going to be seated together all the time. So, one entity for the whole pack of 3 kids plus 1 woman, and 3 entities for the 2nd woman and 2 men. That will be a permutation of 4 out of 4. Now the 3 kids can be seated in 3! ways themselves, and $2 \times 3!$ ways relative to the one woman that is sitting with them (the woman can sit to the right or to the left of the three kids). The woman sitting with kids can be any of the two women, so 2!, and the total number of ways are $4! \times 2 \times 3! \times 2!$.

3. Since we are dealing with passwords, the order matters.

   (a) We want to have at least 2 letters among the 6 digits. So permutation of 2 out of 6 for places for the two letters, $P_2^6$ and there are $26^2$ possible letters for these two places. Then we are left with 4 places that can be filled with any of the letters or digits. So $P_4^4 \times (26 + 10)^4$. So total number of passwords with all these characteristics are $P_2^6 \times 26^2 \times P_4^4 \times 36^4$.

   (b) We want the two first places to always be letters, so 26 options for the 1st place and 26 options for the second place. The rest of the places can be either numbers or letters, so a total of 36 options for each. This will give us a total of $26^2 \times 36^4$.

(c) First place has to be letter, so 26 options. Second place is also a letter, but since repetition is not allowed, we now have 25 options for the 2nd place. Rest of places can be either letters or digits, but we have already used 2 of the 26 possible letters for the first two places, so there are 34 options for the 3rd place, 33 options for the 4th place, 32 options for the 5th place and 31 options for the last place $26 \times 25 \times 34 \times 33 \times 32 \times 31$.

4. Let's count all even integers such that $100 \leq x < 200$. First digit is a 1 and last digit can be 0, 2, 4, 6, or 8, and the middle digit can be any number, so $1 \times 10 \times 5 = 50$. Then for all even integers such that $200 \leq x < 300$ , the total number of cases are $1 \times 10 \times 5 = 50$ and for all even integers such that $300 \leq x < 400$, the total number of cases are $1 \times 10 \times 5 = 50$ plus 1 for the number 400, so total number of even integers greater than 99 and less than or equal to 400 are $50 \times 3 + 1 = 150 + 1 = 151$.

5. Let's count all even integers such that $100 \leq x < 200$. First digit is a 1 and last digit can be 0, 2, or 4, and the middle digit can be any number, so $1 \times 6 \times 3 = 18$. Then for all even integers such that $200 \leq x < 300$ , the total number of cases are $1 \times 6 \times 3 = 18$ and for all even integers such that $300 \leq x < 400$, the total number of cases are $1 \times 6 \times 3 = 18$ plus 1 for the number 400, so total number of even integers greater than 99 and less than or equal to 400 are $3 \times 18 + 1 = 54 + 1 = 55$.

6. We have a total of 20 freshmen, 10 sophomores, and 2 juniors, that are going to be selected with some order for the three medals.

(a) We are interested in the probability that all three winners are from the same class. We only have 2 juniors participating in the competition, therefore probability of having three junior winners is zero, then we have

$$Prob. = \frac{P_3^{20} + P_3^{10} + 0}{P_3^{32}}$$

(b) All juniors should be among winners, and we only have 2 juniors, so the 3rd winner can be a freshman or a sophomore.

$$Prob. = \frac{P_2^3 \times C_2^2 \times P_1^1 \times C_1^{30}}{P_3^{32}}$$

(c) Each one of the three winners is from a different class.

$$Prob. = \frac{P_1^3 \times C_1^{20} \times P_1^2 \times C_1^{10} \times P_1^1 \times C_1^2}{P_3^{32}}$$

7. We have a total of 18 candidates; 10 female and 8 male.

(a) Probability that all 4 elected candidates are female

$$Prob. = \frac{C_4^{10} \times C_0^8}{C_4^{18}}$$

(b) Probability that two men and two women get elected

$$Prob. = \frac{C_2^{10} \times C_2^8}{C_4^{18}}$$

(c) Probability that at least two men get elected is

$$Prob. = \frac{C_2^8 \times C_2^{10} + C_3^8 \times C_1^{10} + C_4^8 \times C_0^{10}}{C_4^{18}}$$

where the first term represents a committee of 2 men and 2 women, the second term represents a committee of 3 men and 1 woman, and the last rem represent a committee of 4 men and no women.

8. The total number of arrangements is $5 \times 4 \times 3 \times 2 \times 1 = 120$, but this isn't quite the right answer. Using the above method, we assumed that all the letters were unique. But they're not! There are 2 Rs, so we're counting every permutation multiple times. So every time we have these 2 permutations: IPROR or IPROR, we actually should have only one permutation: IPROR. Notice that we've over-counted our arrangements by 2! So, the correct answer is $\frac{5!}{2!} = 60$.

9. In the word CORPORATION, we treat the vowels OOAIO as one letter. Thus, we have CRPRTN and (OOAIO). This has 7 (6 + 1) letters of which R occurs 2 times and the rest are different. Number of ways arranging these letters is $\frac{7!}{2!} = 2520$. Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged in $\frac{5!}{3!} = 20$ ways. Required number of ways $= (2520 \times 20) = 50400$.

10. In the word MATHEMATICS, we treat the vowels AEAI as one letter. Thus, we have MTHMTCS and (AEAI). Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice and the rest are different. Number of ways of arranging these letters $= \frac{8!}{2! \times 2!} = 10080$. Now, AEAI has 4 letters in which A occurs 2 times and the rest are different. Number of ways of arranging these letters $= \frac{4!}{2!} = 12$. Required number of words $= (10080 \times 12) = 120960$.

11. Since each desired number is divisible by 5, so we must have 5 at the unit place. So, there is 1 way of doing it. The tens place can now be filled by any of the remaining 5 digits (2, 3, 6, 7, 9). So, there are 5 ways of filling the tens place. The hundreds place can now be filled by any of the remaining 4 digits. So, there are 4 ways of filling it. Required number of numbers $= (1 \times 5 \times 4) = 20$.